Low-Altitude Turbulence Model for Estimating Gust Loads on Aircraft

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The development of a low-altitude turbulence model for estimating aircraft gust loads within the friction layer of the atmosphere is described. The development of the model is based largely on the gust data obtained from the B-66 low-level gust program and climatological wind data provided by the National Weather Records Center, Asheville, N. C. In developing the model, the B-66 vertical gust spectra are used to determine a spectrum function relating spectrum shape to the height above ground and terrain roughness. From the standard deviations of the vertical gust velocities and mean wind speed estimates, empirical standard deviation vs mean wind speed functions are estimated for smooth and rough terrain conditions. Climatological wind data analyses for both smooth and rough terrains are used to provide wind probability density distribution characteristics. From these results, estimates of the standard deviation of vertical gust velocity probability density distribution functions are obtained on the basis of specified confidence limits for the empirical standard deviation gust velocity functions. Finally, the procedure for obtaining probability estimates of the vertical gust velocity is presented for assumed climatic mean wind speed conditions for both smooth and rough terrains. Application of the model to estimating aircraft gust loads is also discussed.

Introduction

ONE of the continuing problems encountered in operating military aircraft in the low-level regime (0 to 1000 ft above terrain heights) is the structural damage sustained from prolonged exposure to atmospheric turbulence. The problem arises because aircraft are being used under conditions for which they were not originally designed, and because current design practice[1] does not account adequately for gust effects at these low altitudes.

This paper is a summary of a study to develop a turbulence model to relate low-level atmospheric turbulence spectra to mean meteorological parameters, terrain conditions, and height. A complete account of this study is available in three Air Force reports.[2-4] This turbulence model will provide a gust load procedure to account for the effects of turbulence on aircraft design load factors and for estimating structure fatigue life. The procedure will require estimating operational histories of aircraft weight, speed, and altitude for the terrain classifications assumed in the model.

General Approach to the Problem

To determine the number of gust load exceedances to be expected for a given operational history of an aircraft, use is made of the mathematical development of Rice[5] of relating spectrum estimates to time history peak values. The procedure is an extension of the application of Rice's result by Press and Steiner[6] to derived gust velocity data in which rms gust velocity probability distributions are specified on the basis of acceleration data obtained from transport operations over a number of airline routes.[7] In the present approach, the root-mean-square gust velocity probability density distributions are determined on the basis of the turbulence model and climatological wind speed distributions. The mean wind speed distribution statistics are obtained from locations representative of terrain conditions investigated in the B-66 program.[8] In the procedure it will be necessary to determine the effect of geographical differences (for similar terrain classifications) on the mean wind statistics.

The basic equation describing the average number of peak values per unit distance exceeding a given load ξ is given by

$$N(\xi) = N_0 \int_0^\infty f(\sigma_w) \exp\left\{\frac{-\xi^2}{2A^2\sigma_w^2}\right\} d\sigma_w \tag{1}$$

where σ_w is the rms of vertical gust velocity, $f(\sigma_w)$ is the probability density function of σ_w, and N_0 and A^2 are defined as

$$N_0 = \frac{1}{2\pi}\left\{\frac{\int_0^\infty \Omega^2|T(i\Omega)|^2\varphi_{w_N}(\Omega)d\Omega}{\int_0^\infty |T(i\Omega)|^2\varphi_{w_N}(\Omega)\, d\Omega}\right\}^{1/2}$$

$$A^2 = \int_0^\infty |T(i\Omega)|^2\, \varphi_{w_N}(\Omega)\, d\Omega$$

where $\varphi_{w_N}(\Omega) = \varphi(\Omega)/\sigma_w^2$ is the normalized spectral density function, and $T(i\Omega)$ is the airplane load response to a sinusoidal gust of unit amplitude.

In general, the behavior of both σ_w and the turbulence spectra are expected to be functions of wind speed and atmospheric stability for given terrain and height conditions. The B-66 spectral data, however, were found effectively independent of both wind speed and stability.[2] It is also convenient to consider the dependence of σ_w on wind speed to be stratified in three atmospheric stability classes. Under these conditions, the average number of load exceedances described by Eq. (1) will depend on height, atmospheric stability classification, and terrain type. The probability density distribution $f(\sigma_w)$ is, in addition, found to depend on the climatic mean wind speed.

Thus, the principal task for establishing a gust load procedure appears twofold: 1) to represent the σ_w probability density function for the required combinations of terrain, height, stability, and climatic mean wind speed and 2) to represent the spectrum function $\varphi_{w_N}(\Omega)$ for varying conditions of terrain and height.

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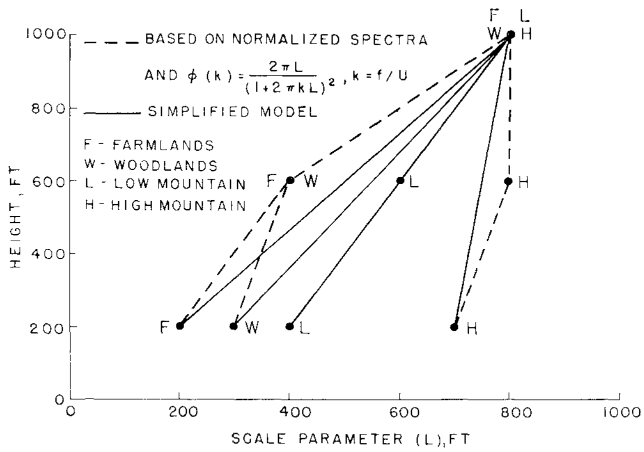

Fig. 1 Scale parameter variation with height and terrain.

Description of Data

The low-altitude gust program conducted by the Douglas Aircraft Company (1958–1960)[8] represents the largest single body of airplane turbulence spectral data available in the height range of 200 to 1000 ft. It is logical therefore to utilize these data as the basis for developing a spectrum model of the turbulent vertical velocity characteristics within the friction layer of the earth's atmosphere. Moreover, since the data were obtained for a variety of terrain conditions, they provide specific examples of the effect on the spectrum for specified heights above the ground.

Although the Douglas data also provide examples of varying atmospheric stability, the number of cases and range of lapse rate conditions represented by the data were not adequate to establish the significance of stability changes on the spectrum shape characteristics. The indication, however, is that the stability effect is no greater than the basic scatter of the data. Since, for a given terrain and height condition this scatter does not appear excessive, it is assumed that the effect of atmospheric stability on the spectrum shape can be neglected.

The procedure for obtaining the vertical velocity estimates from the Douglas B-66 aircraft measurements is represented by the expression

$$w(t) = U\{\alpha(t) - \theta(t)\} + \int n_z(t)\, dt \tag{2}$$

where U is the true speed of the airplane, α the angle of the relative wind obtained from a differential pressure probe arranged to measure flow direction changes, θ the angle of pitch obtained from an attitude gyro (or integrating rate gyro), and n_z the vertical acceleration. The velocity values are analyzed as a function of time and transformed into a space spectrum by the relationship $x = Ut$.

In the B-66 gust program, the data flights were staged from the following Air Force Bases: Edwards, Calif.; Kirtland, N. Mex.; Wright-Patterson (WPAFB), Ohio; and Shaw (SAFB), S. C. Meteorological support for the flights consisted of synoptic surface charts, sky condition reports, and wind data. Airborne temperature soundings were also obtained from the airplane for each set of data flights by descending through at least 1500 ft prior to the data runs and ascending through at least 1500 ft after the last data run. The height of the lowest airborne temperature reading varied with the terrain, but generally was between 50 and 100 ft above the ground. The wind data consisted of pilot balloon observations made in the locality of the data runs, or a reconstruction of the wind speed whenever pibal observations were not obtainable from surface and rawinsonde observations.

Of the eight terrain classifications used in the B-66 program only the low mountains, high mountains, woodlands, and farmlands terrain types have been considered for the present model. For the smooth (farmlands and woodlands) terrains, only the data for WPAFB and SAFB are considered because of the more extensive pibal and other meteorological data available for these localities. For the rougher (low and high mountain) terrains, detailed meteorological data appeared less important; hence, the data obtained at the Edwards and Kirtland bases were combined with the WPAFB and SAFB data for the rough terrain analysis.

Spectrum Shape

The spectrum shape functions describing the characteristics of the B-66 spectral data were obtained in the following manner:

1) Using the integrated spectral density values (spectrum estimate of the square of the standard deviation), the spectra were normalized by dividing the spectrum estimates by the spectrum variance.

2) The spectra were grouped according to terrain and flight level (height above terrain).

3) The high frequency spectral values were considered to approach a -2 power law in accordance with Ref. 9.

4) An empirical function with a single scale parameter was obtained to describe the spectrum variation for each spectral grouping of terrain and height.

5) For each terrain condition the scale parameter was plotted as a function of height and a simplified model providing a linear relationship between scale parameter and height above ground was assumed.

6) Spectrum results reported for other aircraft and tower measurements were used to "test" the spectrum function.

On the basis of the normalized spectra, the spectrum function was represented by

$$\varphi(k) = 2\pi L/(1 + 2\pi kL)^2 \tag{3}$$

where $k = f/U$ is a wave number equal to reciprocal wavelength, f is the frequency in cycles/sec, U is the true airplane speed in ft/sec, and L is a scale parameter in feet related to the terrain roughness and height above ground.

The scale parameter L of Eq. (3) was estimated for the four terrain and three height (200, 600, and 1000 ft) classifications as shown in Fig. 1. According to the estimates shown, the scale parameter variation with height is not necessarily linear, but the assumption of a linear relationship is a convenient one to adopt and is probably within the limits to be expected for the data. Hence, the scale parameter was described by

$$L = h_o + hL_h \tag{4}$$

where the values of L_h and h_o for each terrain are shown in Table 1. In Eq. (4), L is assumed constant above 1000 ft.

In Ref. 3, the representation of the spectrum shape by Eqs. (3) and (4) is compared to normalized spectral envelopes for each of the four terrain classifications at the 200, 600, and 1000-ft levels. Comparisons with other spectral data obtained from aircraft[10–12] and tower[10,13,14] observations are also shown in Ref. 3. All of these data are summarized in Fig. 2 by presenting the spectral data as a function of nondimensional frequency kL, where k is the frequency divided by true aircraft speed (or wind speed for tower data), and L is the scale parameter. For the tower data shown in Fig. 2, L is assumed equal to the height above ground in accordance with meteorological practice.[13,14]

Table 1 Values of L_h and h_o for each terrain

Terrain class	L_h	h_o
F	0.75	50
W	0.625	175
L	0.5	300
H	0.125	675

The Dryden[15] spectrum form shown in Fig. 2 is based on wind tunnel measurements of turbulence in the streamwise direction produced by grids placed in the tunnel normal to the mean flow. Dryden obtains the spectrum form by fitting the computed autocorrelations with exponential curves. The L values are then obtained from

$$L = \int_0^\infty R(x)\, dx$$

where $R(x) = (1 - x/2a)e^{-x/a}$ (a is an arbitrary constant).

Vertical Velocity RMS Behavior with Wind Speed for Smooth and Rough Terrains

Smooth Terrain Estimates

Mean wind speed and lapse rate values needed to estimate the behavior of the B-66 standard deviations of vertical velocity as a function of wind speed for varying atmospheric stability conditions for the smooth terrain classifications are reported in Ref. 3. These wind speed and lapse rate values are revised estimates obtained from the original B-66 data. The procedure for obtaining these estimates, as explained in Ref. 2, was the following: 1) the wind speed estimates of Ref. 8 were reviewed and modified, when necessary, to provide general agreement with synoptic wind fields reported before and after each flight and 2) the flight temperature and lapse rate readings for each of the flights were compared with synoptic data and temperature soundings from radiosonde stations in the vicinity of each flight to provide a basis for estimating the temperature differences between the surface and 1000 ft.

Based on the revised estimates, three functions for the standard deviation of vertical velocity were obtained for wind speeds subclassified according to the following atmospheric stability groups: stable ($0°F < T_{sfc} - T_{1000} < 5°F$), near adiabatic ($5°F \leq T_{sfc} - T_{1000} < 6°F$), and unstable ($T_{sfc} - T_{1000} \geq 6°F$).

Although both the WPAFB and SAFB data were considered in the analysis, it was found that the Shaw data, for comparable terrain conditions, did not change the σ_w relationship with wind speed for the unstable lapse rate case. For some presently unexplained reason, however, the σ_w values for the near adiabatic temperature gradient data appear consistently higher for Shaw than for the Wright-Patterson estimates. Since the reason for this is not known at present,† only the Wright-Patterson data are considered in this paper.

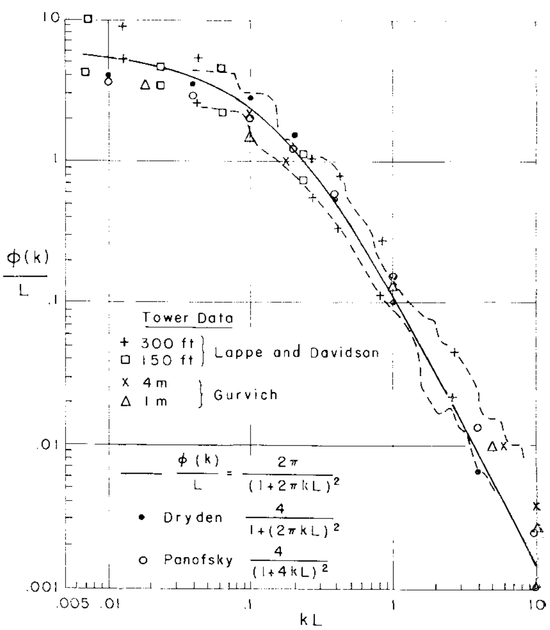

Fig. 2 Comparison of airplane and tower spectrum measurements of turbulence plotted nondimensionally. The dashed lines represent envelope of airplane spectra.

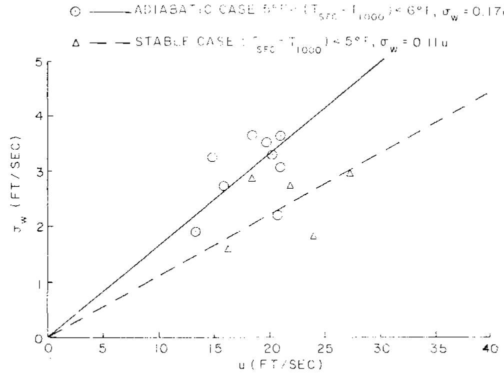

Fig. 3 Standard deviation of vertical velocity as a function of wind speed for adiabatic and stable lapse rate conditions.

For the three atmospheric stability classes, graphs of σ_w (the standard deviation of vertical velocity) as a function of u (mean wind speed) were made on the basis of the revised estimates. These are shown in Figs. 3 and 4. The data points represent average values for the runs in each of the flights. By combining the data runs into single estimates for each flight, it is assumed that the effect of height (between 200 and 1000 ft) on the σ_w estimates is negligible. Under stable and near adiabatic conditions this is in accordance with Refs. 2 and 16. Under unstable conditions, however, Refs. 2 and 13 report an increase of σ_w with height. An increase of about 8% in σ_w was found in Ref. 2, as the height increased from 200 to 1000 ft. For the present, however, this has been neglected, since the results would not be significantly affected by changes of this magnitude.

A least squares procedure was used to fit the standard deviation vs wind speed data. In the case of both the stable and near adiabatic cases (Fig. 3), the lack of data at very low or very high wind speeds necessitated constraining the "line of best fit" to pass through the origin. This constraint appears reasonable from both an empirical and theoretical viewpoint (e.g., Pasquill).[17] Linear correlation coefficients for these two cases are 0.37 and 0.48, respectively. These values are quite low, and our confidence in the specific values given for the regression coefficients cannot be very great. Statistically there is about a 5% chance that these constants would be more than twice as great. There is, however, some additional measure of confidence to be gained from similar analyses of tower data by Panofsky[13] and Singer[16] who report regression coefficients of similar magnitudes.

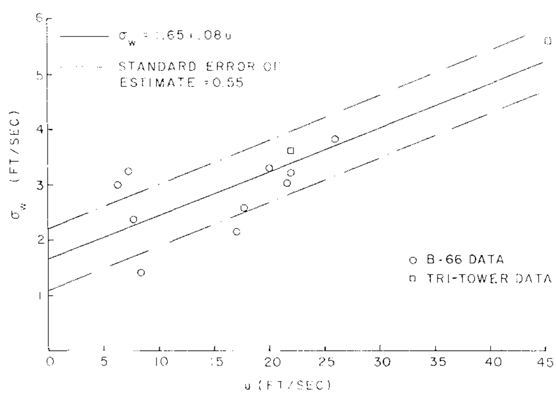

Fig. 4 Standard deviation of vertical velocity as a function of wind speed for unstable lapse rates.

† One possibility is that the estimated lapse rates are slightly too low, since the discrepant standard deviation values would agree with the unstable data.

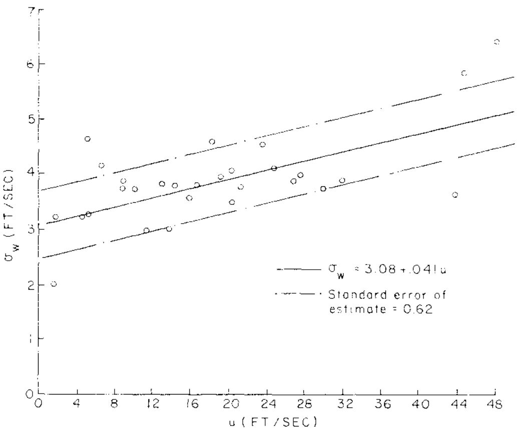

Fig. 5 Standard deviation of vertical velocity vs wind speed for low mountain terrains.

In the case of unstable lapse rates, Fig. 4 indicates a more gradual increase of σ_w with increasing wind speed. Although several comparatively low wind speed flight runs were obtained with the B-66 during unstable conditions, no flight data were obtained during very high wind speed conditions. For this reason the tri-tower σ_w data[10] (obtained under relatively strong wind speed and thermal instability conditions) have been added to the B-66 data in Fig. 4. For this comparison it is assumed that the terrain surrounding the Brookhaven site can be considered similar to the B-66 farmlands classification.

Using the least squares procedure for the unstable data, the following regression line was obtained:

$$\sigma_w = 1.67 + 0.08\,u \tag{5}$$

In this case the linear correlation coefficient is 0.82, and 95% confidence limits for both constants of Eq. (5) are about ±50%. Nine flights are represented by these data.

Rough Terrain Estimates

Over mountainous terrain, the effect of atmospheric stability on the σ_w estimates does not appear significant. This conclusion is based on the revised estimates for WPAFB and SAFB which indicate a nearly equal partition of near adiabatic and unstable lapse rate values for the low mountain terrains. For these data, no significant stability behavior was observed when σ_w was plotted as a function of wind speed. In fact, when these values were combined with the Edwards and Kirtland low mountain data,[4] the standard deviations indicated a roughly linear increase with wind speed rather similar to the unstable smooth terrain data. The estimates are shown in Fig. 5.

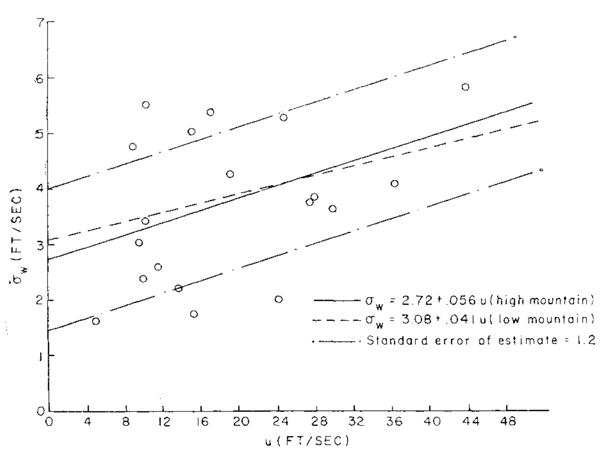

Fig. 6 Standard deviation of vertical velocity vs wind speed for high mountain terrains.

A least squares linear regression for the data of Fig. 5 provided

$$\sigma_w = 3.08 + 0.041\,u \tag{6}$$

The linear correlation coefficient is 0.69, and the 95% confidence limits for the regression line constants are about ±42%, assuming each of the 29 flights for the low mountain data as representing one sample.

A similar analysis for the high mountain σ_w values indicates considerable scatter about the low mountain regression line, as shown in Fig. 6. The least squares regression line for these data,

$$\sigma_w = 2.72 + 0.056\,u \tag{7}$$

is seen to correspond rather closely to Eq. (6) for the low mountains.

The scatter of the high mountain data resulted in a linear correlation coefficient value of 0.49. The 95% confidence limits for the 19 flights represented by these data are about ±95%.

The notable increase in the scatter of the high mountain σ_w estimates is believed to be the direct consequence of the increase in terrain roughness as compared to either the low mountain or smooth terrains. This behavior of the σ_w estimates for very abrupt terrain changes was noted in the tri-tower experiment,[10] wherein it was observed that variances computed from time histories corresponding to 1-mile segments reflected the principal underlying terrain features.

Wind Profile Behavior

To estimate the mean wind speeds at flight levels from observations taken near the surface under varying conditions of thermal stability, wind speed observations from a 1400-ft television tower at Cedar Hill, Texas were studied in detail. The results of this analysis, reported in Refs. 3 and 18, provide wind profile characteristics for heights of 30 to 1300 ft, for lapse rate conditions ranging from near isothermal to superadiabatic.

The principal results of this study are threefold:

1) Under near adiabatic to superadiabatic conditions the log profile law ($u/u_1 \ \alpha \ \log z/z_1$, where u_1 is the wind speed at reference height z_1) describes well the wind speeds to a height of 450 ft. Above this height the winds remain substantially constant.

2) As the lapse rate becomes more stable, the form of the wind profile depends critically on the lapse rate structure. These profiles can be represented by a power law of the form $u/u_1 \ \alpha (z/z_1)^p$. In general, as the lapse rate changes from near adiabatic to isothermal, p decreases in a near linear fashion. The variation of p with lapse rate is approximated by $p = 0.43 - .05\,\Delta T$ for $2 < \Delta T < 5$ where ΔT is in °F/10^3 ft.

3) As the lapse rates approach isothermal and inversion values, turbulence intensities diminish rapidly; for this reason it is not necessary to consider these profiles in describing σ_w as a function of wind speed.

Climatological Wind Speed Statistics for Smooth and Rough Terrain

Smooth Terrain

A description of wind speed distribution characteristics for smooth terrain conditions are presented in Ref. 3 on the basis of radiosonde and pibal observations obtained at Dayton, Ohio for the two year period of June 1956 to May 1958. In the analysis the following factors were considered: 1) atmospheric stability, 2) seasons, 3) time of day, and 4) height.

The result of the analysis for the two year period at Dayton indicated that the probability distributions for all of the conditions examined could be approximated by a single probability function using dimensionless wind speed $(u/\bar{u})$ as the independent variable, where $\bar{u}$ is the average mean wind speed, hereinafter referred to as "climatic mean wind." This suggested that if the $u/\bar{u}$ probability distribution behavior were known for given geographical areas, the actual wind speed frequency distribution could be specified from a knowledge of the climatic mean wind.

To test this probability function for a larger sample and to compare it with another smooth terrain location, additional data were obtained for June 1953 through May 1957 for 1000 EST and 1600 EST at Dayton, and for 1500 CST at Fort Worth, Texas. The results obtained for these data corroborated the earlier study. Figure 7 is a summary of all of the smooth terrain data subclassified according to the four factors listed previously. It is evident from this figure that both Dayton and Fort Worth can be reasonably represented by a single probability function.

Although no completely satisfactory analytical representation has yet been found for the density function, the form suggested by Ref. 3 for the cumulative probability may prove generally satisfactory. The probability density form corresponding to the cumulative distribution form of Ref. 3 is

$$p(x) = a\,b\,x^{b-1}e^{-a\,x^b} \tag{8}$$

where $x = u/\bar{u}$; and a, b are arbitrary parameters. This form is similar to the distribution first suggested by Weibull[19] to describe the observed behavior of many physical randum variables. In this connection, it is interesting to note that Gumbel[20] has derived the Weibull distribution in terms of extreme value theory.

Rough Terrain

Initially, wind speed data were obtained for Medford, Ore. for the same stability and height classifications as Dayton and Fort Worth. Analysis of these data indicated that near the surface the statistical distribution functions varied considerably with atmospheric stability and the number of times the winds were below threshold values (near calm). At the higher levels, however, the wind distribution characteristics appeared somewhat similar to the smooth terrain results.

Because of the current practice of making soundings twice daily (1200 and 2400 GMT) the available stations reporting daytime observations were limited to the western half of the country. Beside Medford, therefore, the stations selected for the rough terrain analysis included Las Vegas, Winnemucca, and Ely, Nev. and Boise, Idaho. (The terrain characteristics surrounding each station are described in Ref. 4.)

Since the σ_w vs wind speed data for the rough terrains indicated that σ_w could be assumed independent of lapse rate conditions, the wind data were not subclassified with stability. Wind direction, however, was included in the data and the height range was extended to 5000 m above the surface.

The wind statistics indicate that at or above 300 m the wind speed distributions are fairly uniform; the principal affect of wind direction is on the climatic mean wind speed. This is shown in Fig. 8, where the probability density values of $u/\bar{u}$ have been plotted for the five rough terrain stations, for heights varying from 300 m to nearly 5000 m above the surface. The tagged circle and triangle symbols in the figure represent the 300 m Las Vegas level and most of the higher levels of Boise, respectively. These data do not agree with the much larger number of other observations. The solid curves in Figs. 7 and 8 represent the estimated probability density distribution behavior of $u/\bar{u}$ for smooth and rough terrains. The dashed curves are approximations provided by Eq. (8).

In view of these results, it appears reasonable to consider generalizing wind speed probability density distributions of $u/\bar{u}$ for both smooth and rough terrains. In the case of smooth terrains, a single function would appear satisfactory for the surface as well as the higher levels; whereas, for the rough terrain, the surface distributions appear too erratic to include in the over-all representation by a single function. Since, however, the present turbulence model is unlikely to

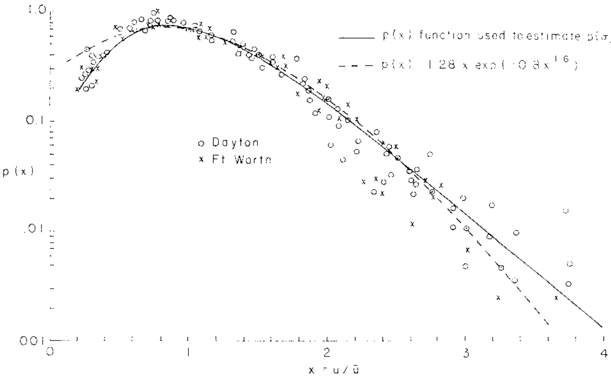

Fig. 7 Wind speed probability densities for smooth terrain stations.

be concerned with wind distribution functions very close to the ground in rough terrain localities, a single analytic rough terrain wind distribution function is also postulated.

Over smooth terrain it is hypothesized that the climatic mean wind near the surface will be sufficient to permit determining (with the wind profile characteristics) the climatic mean wind at other levels within the surface friction layer (first several thousand feet). Over rough terrain, however, it is necessary to establish, if possible, the approximate behavior of the wind speed with height. To this end, the climatic mean wind speed variation with height was examined for each rough terrain location. The climatic mean wind speed ratios $\bar{u}_z/\bar{u}_{sfc}$ are shown in Fig. 9 as a function of height above the surface station. As a rough approximation to the behavior of this wind speed ratio with height, the function

$$\bar{u}/\bar{u}_{sfc} = 1 + 8 \times 10^{-5}\,z^{1.2} \tag{9}$$

is shown in Fig. 9.

Use of Turbulence Model to Estimate Gust Velocity Probabilities

On the basis of the foregoing results, probability estimates of exceeding specified vertical gust velocity values are now presented. This estimate is based on: 1) assumed climatic mean wind speed values, 2) an assumed allocation with respect to rough and smooth terrains and atmospheric stability conditions, 3) a single height condition of 1000 ft, and 4) the use of the σ_w vs wind speed relations with 95% confidence limits assumed for the regression constants to account for probable extreme values of σ_w about the given regression lines of Figs. 3–6.

Procedure Used for Estimating Gust Velocity Probabilities

In order to simplify the calculations as much as possible the number of conditions selected for analysis were held to a

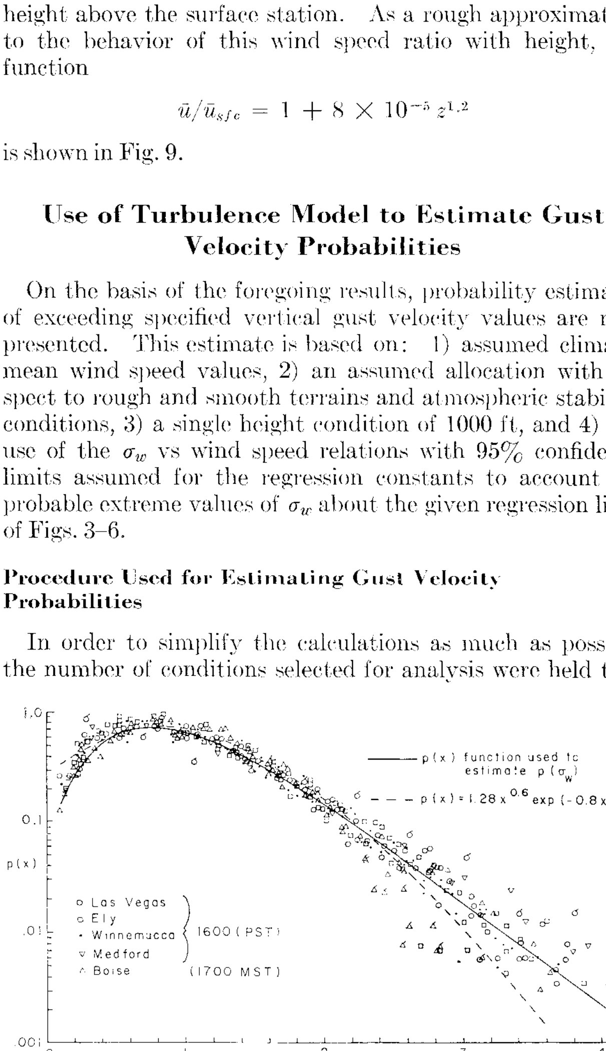

Fig. 8 Wind speed probability densities for five rough terrain stations for heights of 300 to 4340 m above surface stations.

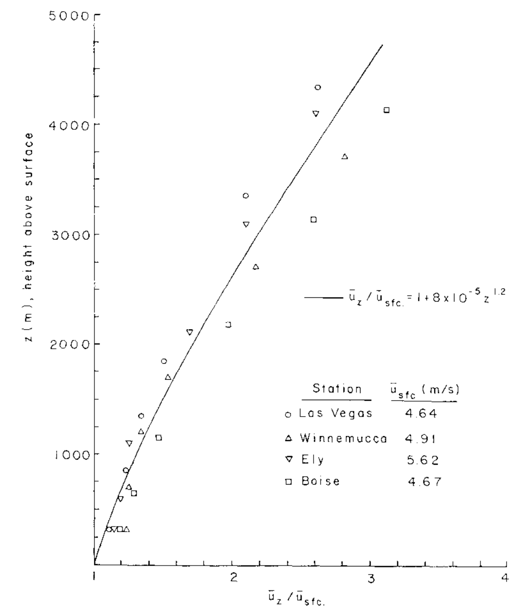

Fig. 9 Ratio of climatic mean wind speed at a height z to the surface value a as function of the height.

minimum. The terrain conditions were divided equally between smooth and rough terrains. The rough terrain was apportioned 60% high mountain and 40% low mountain. The smooth terrain conditions were apportioned 60% unstable and 40% stable. The climatic mean wind values selected were based on the climatological data reported previously. These are $\bar{u} = 10$ m/s for the rough terrain and $\bar{u} = 7$m/s and 10 m/s for the unstable and stable lapse rate conditions, respectively, for the smooth terrain.

Based on these assumptions, the following σ_w relations were obtained:

$$\left.\begin{array}{lll} \sigma_w = 1.6 + 1.1\chi & \text{(high mountain)} \\ \sigma_w = 1.3 + 0.6\chi & \text{(low mountain)} \\ \sigma_w = 0.72 + 0.77\chi & \text{(unstable)} \\ \sigma_w = 1.6\chi & \text{(stable)} \end{array}\right\} \quad (10)$$

where $\chi = u/\bar{u}$ and σ_w is expressed in m/s.

Using the relations given by Eq. (10) and the probability density distributions of $u/\bar{u}$ described in Figs. 7 and 8, the probability density distribution estimates of σ_w shown in Fig. 10 were obtained.‡ For each condition of Eq. (10), the expected number of exceedances of vertical gust velocity was determined from the expression

$$\frac{N(w)}{N_0} = \tau \int_0^\infty f(\sigma_w) \exp\left\{\frac{-w^2}{2\sigma_w{}^2}\right\} d\sigma_w \qquad (11)$$

where τ is the relative exposure to each σ_w probability density distribution. Summing Eq. (11) for the exposure assumed for Eq. (10) provides the expected probability of exceeding a given vertical gust velocity.

When the procedure is used to estimate aircraft load exceedances the number of environmental and aircraft configuration conditions must of course be sufficient to provide a realistic representation of the load experience. In this case, the summation of Eq. (11) for a given terrain classification could be expected to involve subclassifying with respect to such factors as aircraft speed and height, time of day, and possibly seasonal climatic mean wind speed for particular geographical regions.

In Fig. 11, the result of applying Eq. (11) to each condition assuming $\tau = 1(100\%$ exposure) is shown, along with the result for the actual exposure history assumed.

‡ Note that the probability density estimates can, unlike the cumulative probability estimates, be greater than one. The required condition to be satisfied by the probability density is

$$\int_0^\infty p(x)\,dx = 1$$

Comparison with Current Procedures

In order to provide some comparison of the gust velocity exceedance probabilities estimated previously with current practice,[1,6] the following estimates were made. In Ref. 6 an expression for the probability density function of σ_w is given by

$$f(\sigma_w) = -(1/b_1)(2/\pi)^{1/2} \exp\{-\sigma_w{}^2/2b_1{}^2\} \qquad (12)$$

where b_1 is a scale parameter proportional to $\sigma_w/\sigma_{u_{de}}$. The ratio is taken as 1.77 in Ref. 6 in the 0 to 2000-ft height range. The b_1 scale parameter corresponding to this $\sigma_w/\sigma_{u_{de}}$ value is 4.6. In Ref. 1 the b_1 value has been reduced to 3.9 for the 0 to 1000-ft height range.

To estimate the $\sigma_w/\sigma_{u_{de}}$ ratio for the B-66 program, ten of the flights used in the σ_w vs wind speed analysis, for which both σ_w and $\sigma_{u_{de}}$ estimates were available, were examined. The analysis showed that the $\sigma_w/\sigma_{u_{de}}$ ratio ranged from 0.66 to 0.9 for the ten flights, with an average value of 0.76. The average value provided a b_1 estimate equal to about 2. In Figs. 10 and 11, distribution curves corresponding to b_1 values of 2 and 3.9 are shown.

Discussion of Control Motion Effects on the Estimates

The rather large variability and comparatively low values of the B-66 $\sigma_w/\sigma_{u_{de}}$ ratios are believed to be a direct consequence of small, but significant, pilot control inputs. Although these control movements (elevator deflections in this case) are cancelled by the method used to determine the gust velocities, they remain in the acceleration measurements used to determine u_{de}.

If, as appears from the B-66 data, these control corrections constitute a significant factor in the aircraft loads environment, then provision should be made to record these control motions and to incorporate them in the present gust load model. The basis for doing this is outlined in Ref. 4. In essence, this would involve generalizing for each aircraft (and for each response quantity) an increment to be added to σ_w as an additional σ_w experienced by the aircraft as the result of operating in a particular turbulence environment. The expression for the effective rms gust velocity is

$$\sigma_{we} = \sigma_w \left\{1 + \left[\int_0^\infty \frac{\varphi_{\delta e}(\Omega)}{\varphi_w(\Omega)}\, \varphi_{w_N}(\Omega)|H_{y_{\delta e}}(i\Omega)|^2\, d\Omega\right]\right\} \qquad (13)$$

where $\varphi_{\delta e}(\Omega)$ and $\varphi_w(\Omega)$ represent the elevator displacement and gust velocity spectra; $\varphi_{w_N}(\Omega)$, the normalized gust spectra [Eq. (3)]; and $H_{y_{\delta e}}(i\Omega)$ the transfer function of any aircraft response item y to elevator input δ_e.

Discussion of the Turbulence Model

In developing the present turbulence model two principal questions arise: one concerning the adequacy of the data collected during the B-66 low level gusts program; the other

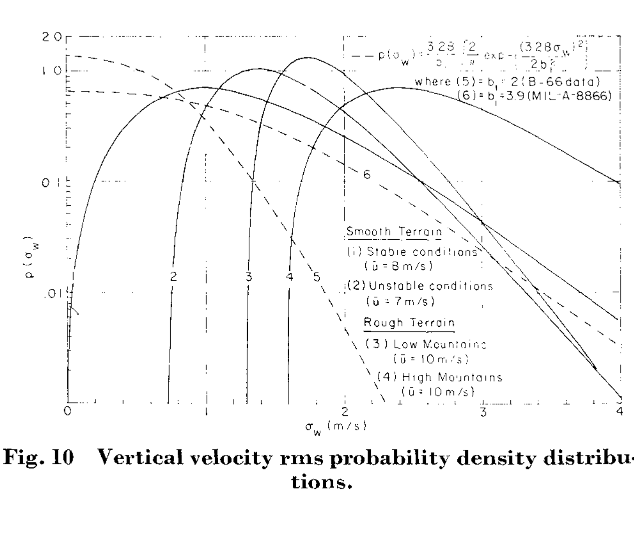

Fig. 10 Vertical velocity rms probability density distributions.

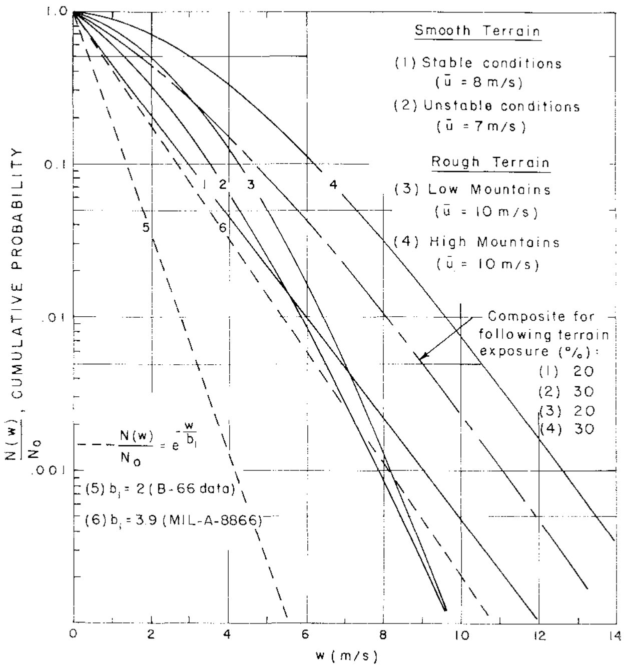

Fig. 11 Vertical velocity cumulative probability distributions.

concerning the description of the behavior of turbulence in terms of the mean meteorological quantities. Undoubtedly, the most serious shortcoming of the B-66 program was the inability to obtain airborne mean wind measurements. This is probably most significant for the rough terrain data, since the wind estimates for these conditions are more likely to be in error. As discussed previously, however, the data in Figs. 5 and 6 suggest that the biggest factor contributing to the scatter of the σ_w estimates for mountainous terrain is probably terrain induced inhomogeneity. Hence, even if the wind conditions were known perfectly, the standard deviation estimates could be expected to vary considerably, depending on the underlying terrain.

Although the rough terrain analysis is thus complicated by the terrain associated σ_w variability, it is otherwise simplified by the fact that the lapse rate conditions apparently need not be considered. Thus, a study of climatic mean wind behavior as a function of geographical location can be expected to provide the basic information needed with the σ_w estimates for the rough terrain part of the gust load model.

Over smooth terrains, the climatic mean wind is related to lapse rate conditions, time of day, seasons, and, of course, height. Therefore, a more careful study of these factors as a function of geographical location is necessary to complete the smooth terrain portion of the gust load model.

The question of accuracy of the gust velocity estimates obtained in the B-66 program cannot be answered directly. However, since the spectrum characteristics (Fig. 2) and the σ_w vs wind speed relations (Figs. 3 and 4) appear to be in general agreement with tower measurements, there is reason to believe that the B-66 measurements were at least adequate for the initial effort of establishing approximate relations between the turbulence and the mean meteorological quantities. Over smooth terrain there is the practical possibility of refining these estimates by a carefully worked out tower-airplane program.

This is not the case over rough terrain, where towers would prove of only limited usefulness. Under these conditions a reliance on the use of specified confidence limits, as applied to available rough terrain flight measurements of σ_w, may prove to be the only practical approach.

Conclusions

Vertical velocity gust spectra, mean wind speed, and lapse rate data obtained from the B-66 low-level gust program have been used to develop a low-level turbulence model. Both the turbulence spectra and rms gust velocity vs mean wind speed characteristics indicate reasonable agreement with independent meteorological observations.

Although the formulation of the turbulence model appears reasonable, it is evident that there is a definite need for additional data to verify the characteristics of the model. In particular, the behavior of the rms gust velocity as a function of wind speed for varying atmospheric stability conditions should be established for a much broader range of wind speed conditions. For this purpose a tower-airplane program is believed indispensable.

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